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Capacity allocation and downsizing decisions in project portfolio management

This paper aims to gain insight into capacity allocation and downsizing decisions in project portfolio management. By downsizing, we mean reducing the scale or size of a project and thereby changing the project's content. We first determine the amount of critical capacity that is optimally allocated to strategic projects with deterministic or stochastic workloads for a single-period problem when the impact of downsizing is known. In order to solve the multi-period problem, we have modeled the behavior of the portfolio in subsequent periods as a single project for which the return on investment can be estimated. Secondly, we investigate how the scarcity of resources affects the (expected) value of projects. The *independent* (expected) project value is calculated under the assumption of unlimited capacity; in contrast, the *dependent* (expected) project value incorporates the resource constraints. We find that the dependent project value is equal to the independent project value when the return on investment of the portfolio is sufficiently low. In addition, we determine the relation between the return on investment of the portfolio and the value of a project and conclude that the impact of resource scarcity on the value of a project cannot be fully captured by the common financial practice of adapting the discount rate with the estimated return on investment.

Keywords: project portfolio management; downsizing; stochastic workload.

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1. Introduction

Financial models such as the Net Present Value (NPV) model, generally assume that resources are available in unlimited supply, be it at a certain cost. Consequently, a project's NPV is computed by discounting its cash flows at the opportunity cost of capital (Brealey & Myers 2003). Accordingly, Loch & Bode-Greuel (2001) suggest to incorporate the scarcity of resources by adapting the discount rate with the average return on investment. In this paper, we investigate how the scarcity of resources affects the value of projects. We speak of the *independent* (expected) project value when it is calculated under the assumption of unlimited capacity. In contrast, the *dependent* (expected) project value incorporates the resource constraints. We find that the dependent project value is equal to the independent value when the return on investment (ROI) of the portfolio is sufficiently low. In addition, we determine the relation between the ROI of the portfolio and the value of a project and conclude that the impact of resource scarcity on the value of a project cannot be fully captured by the common financial practice of adapting the discount rate with the estimated ROI.

This paper considers environments where capacity expansion through the deployment of additional resources is not possible, such as, for instance, most R&D (Research and Development) or NPD (New Product Development) departments, where the number of researchers or other critical resources is fixed as a result of a strategic decision depending on the revenue stream and costs. The company's restricted resource availability then represents the most important constraint on project selection.

We consider the case of *static* project selection, where the set of projects available for execution during the planning horizon is known in advance. Such static models are mainly suitable for the selection of *internal* projects, which are projects that have been proposed by internal customers. Since projects may have uncertain requirements for renewable resources, which are available in limited amounts, resources may become overcommitted. Evidence from practice as well as from literature suggests that when extra resources cannot be acquired, several options exist: reducing the scale or size of some strategic projects, reallocating resources, slowing down the execution, etc. In this paper, we investigate the *capacity allocation* and *downsizing* decisions in *project portfolio management*. Project portfolio management deals with the continuous flow of projects; it entails choosing the right projects and the associated capacity allocation. Such decisions are typically subject to periodic revisions (e.g. on a half-yearly basis). By downsizing, we mean reducing the scale or size of a project and thereby changing the project's content to the extent that the project's capacity requirements comply with the allocated capacity. Downsizing is based on the principle that project selection is not an all-or-nothing decision, but that multiple funding alternatives exist for executing a project (Sharpe & Keelin (1998); Kavadias & Loch (2004)). In this paper, we determine the effect of downsizing on a project's value and the circumstances under which downsizing is recommendable.

We focus on the selection of *strategic projects*, which are essential to guarantee the future profits of the company. This type of projects contrasts strongly with *utility projects*, which are low-risk projects that can easily be cut or postponed without any impact on the attainment of the strategic goals of the company. If we evaluate the performance of strategic and utility projects by the same measures, utility projects might never get selected (Levine 2005). It is therefore advised to reserve parts of the total capacity for each project type (Cooper et al. 1998) and to divide the resource budget into more focused budgets or *strategic buckets* (Chao & Kavadias 2008). For this reason, we consider the selection and capacity allocation of strategic projects for a fixed amount of available capacity. We examine the case where a company has a flexible renewable resource pool with more or less equal competencies present within the different departments. This allows us to consider only one renewable resource type. The accuracy of the decisions made, however, increases when we make a finer distinction between resource types by considering different functions such as manager, project engineer and technician; this extension will be presented at the end of the paper.

A project's revenue depends on its characteristics and its allocated capacity. The main project parameters are overall value, workload and *downsizeability*. The overall value is determined by the strategic and the financial project value; it incorporates the project risk and its maximization is considered to be the primary objective. The workload is estimated in terms of the number of manhours of each resource type required during every period of the problem horizon and may be either deterministic or stochastic. In the deterministic case, the allocated capacity fixes the scale of the project before the start of the execution. In case of stochastic workloads, the project is only downscaled if the actual workload exceeds the allotted capacity, in order to respect the resource limits. Reducing the scale or size of a project equates with changing its content to comply with the allocated resources. As an example, in a drug development process for cancer treatments, a downsize could result in dropping one of two alternative product forms (intravenous and oral) in one of two markets (tumor types A and B) (Sharpe & Keelin 1998). The downsizeability expresses the effect of such a scale reduction on the overall value of the project.

This paper aims to gain insights into the capacity allocation and downsizing decisions in project portfolio management; its focus is twofold. First, we determine the amount of critical capacity that is optimally allocated to strategic projects with deterministic or stochastic workloads for a single-period problem when the impact of downsizing is known. In order to solve the multi-period problem, we model the behavior of the portfolio in subsequent periods as a single project for which the return on investment (ROI) can be estimated. The ROI is variable since it results from the events in previous periods. Under uncertainty, precommitment to any action is not necessarily optimal, so that the NPV rule is no longer appropriate. We therefore turn to real option analysis (ROA) (cfr. Dixit & Pindyck (1994), Trigeorgis (1997)) to correctly incorporate the value of managerial flexibility in the portfolio appreciation. Secondly, as already mentioned, we study the relation between the (expected) project value and the scarcity of resources.

This paper is organized as follows. Section 2 surveys the literature that is most relevant to our problem. In Section 3, an extensive problem formulation is provided. Subsequently, we discuss the impact of downsizing on a strategic project's value in Section 4. In Section 5, we solve the selection and capacity allocation problem for multiple strategic projects, both in case of deterministic as well as stochastic workloads. The behavior of the single-period project portfolio is studied in Section 6, and our findings allow us to extend the problem horizon to multiple periods and to determine the value of multi-stage projects in Section 7. Model extensions such as the downsizing of multiple stages and multiple resource types are considered in Section 8. Section 9 contains some conclusions.

2. Literature

Our work adheres to different research domains, one of which is R&D portfolio management (Martino (1995), Cooper et al. (1998), Kavadias & Loch (2004) and Loch et al. (2006)). Kavadias & Loch (2004) assume that projects are never canceled for budgetary reasons, and estimate the project values independently, so without incorporating the resource requirements of the other projects in the portfolio. A project is then characterized by its first-year capacity need and the value of the option to continue the project execution at the end of this period. In the current paper, we consider the case where resources are shared and limited, so that the option values of the projects cannot be regarded as independent, but rather depend on the other projects in the portfolio and their characteristics, as well as on future opportunities. This view is shared by Girotra et al. (2007), who claim that project interactions in a multi-project context significantly alter the value of a project. More specifically, they investigate how the presence of other projects in a portfolio influences a project's value, e.g. when another project fails, this frees up resources for remaining projects in the portfolio. The authors make an event study around the failure of stage-based pharmaceutical projects and their effect on the market valuation of the company.

Loch & Bode-Greuel (2001) calculate the option value of projects that compete for the same resources by using a discount rate that reflects the average return on investment of all R&D projects. Huchzermeier & Loch (2001) apply a real-options approach to assess the value of flexibility for a single R&D project and they investigate the impact of five different types of variability on the option value. Variability that issues from the shared resource usage in a multi-project environment, however, is not taken into account. This subject has been studied by Ding & Eliashberg (2002), who construct optimal NPD pipelines and analyze the influence of variability in the available resources (caused by the limited probability of survival of multi-staged NPD projects) on the selection process. In our model, we assume that insufficient capacity leads to a downsizing of projects. The value of downsizeable projects cannot be straightforwardly determined, since it depends on the available capacity as well as on the characteristics and the downsizing capabilities of all other projects in the portfolio. The study of interdependent options is still very limited, for an overview we refer to Childs et al. (1998).

Research on portfolio management and resource allocation has brought about other interesting surveys, among which a study by Heidenberger (1996). The author presents a mixed integer linear programming (MILP) model for solving the project selection and funding problem. He considers multiple scarce resources and assumes that their input affects the probability of project success. Lockett & Gear (1973) determine the portfolio composition through simulation of the resource requirements of different projects, starting from decision trees. In Loch & Kavadias (2002), marginal returns are used to optimally split a scarce budget between NPD programs over multiple periods while decisions have multiperiod consequences. In their model, the NPD programs are assumed to all have either increasing or decreasing returns, while the resource allocation to a single program is only bounded by the available budget. These NPD programs may be considered as special cases of the strategic projects dealt with in our paper. Our models, however, do allow for portfolios to contain both projects with increasing and with decreasing returns and impose lower and upper bounds on the projects' resource allocation.

This paper is also closely related to the project planning literature. A great deal of this literature is dedicated to the selection and sequencing of activities in order to maximize the NPV (De et al. (1993), Gupta et al. (1992) and Kyparisis et al. (1996)). This problem is extended to the selection of R&D activities from a set of alternatives by Granot & Zuckerman (1991). A more detailed planning is performed by Kis (2005), and by Kolisch & Meyer (2006) for pharmaceutical research projects; they model the problem of selecting and planning projects as extensions of the resource-constrained project scheduling problem (RCPSP) (cfr. Demeulemeester & Herroelen (2002); Neumann et al. (2002)).

Downsizing in the deterministic sense (i.e. where we decide upon the project's scale before its execution starts) has barely been touched on in the existing literature. Sharpe & Keelin (1998) discuss how considering downscaling and upscaling projects can enlarge the set of projects available for selection and in this way they succeed in increasing the company's output. Huchzermeier & Loch (2001) derive the value of an option to decrease or increase

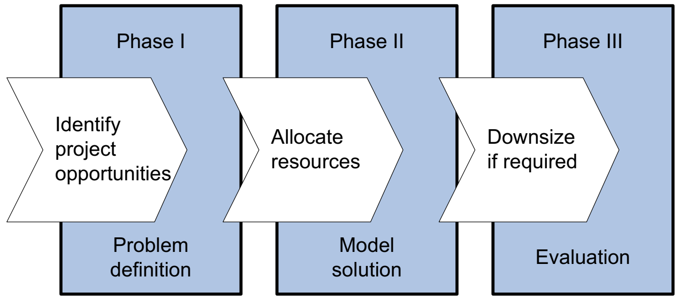


Figure 1: The project selection and capacity allocation process for downsizeable projects.

the project scale depending on the development of the market but do not consider a multi-project environment where resources are shared. Implicitly, the idea of downsizing has been used by Kavadias & Loch (2004), who allow for a reduction of a project's resources down to a lower feasibility limit. Between this lower bound and the upper feasibility limit the project return function is concave. Bayus (1997) investigates the market conditions under which it is optimal to enter the market with a low-performance project (this is a product with reduced features, which is equivalent to a downsized project), rather than delay the time-to-market until a high-performance product has been developed. To the best of our knowledge, the problem of downsizing a project with stochastic workloads has not been studied in literature. An important contribution of our paper lies in the modelling of downsizing decisions both for the case of deterministic as well as for stochastic workloads in an environment with limited available resources.

3. The project selection and capacity allocation process

The selection and allocation process comprises three phases, visualized in Figure 1. During the first phase, discussed in Section 3.1, we identify the strategic project opportunities available for execution. Next, the resources are split between the selected projects (Section 3.2). In Phase III, we downsize running projects that exceed their allocated resources. The downscaling process is the topic of Section 3.3.

3.1 Phase I: Identify project opportunities

Before selecting projects and allocating resources, a company should identify all project opportunities. A strategic project's expected output is determined by its expected commercial value (ECV), workload, downsizeability and allocated capacity. The ECV is usually based on a decision-tree analysis and incorporates the future stream of earnings from the project, the commercialization and development costs, along with both the market and technological uncertainty involved in the development of the project. It also considers the strategic importance of the project (Cooper et al. 1998).

A project's workload is expressed as the number of manhours the project requires during the problem horizon. The workload of a strategic project may be either deterministic or stochastic; in the stochastic case, the workload P is a random variable, with mean μ. When the workload is considered to be deterministic, P is replaced by μ. The resource budget available for strategic projects is expressed as the available number M of manhours within each period of the planning horizon. Value $\hat{M}$ ($\hat{M} \leq M$) indicates the amount of capacity allotted to a strategic project. We define the scaled input I as the apportioned project's workload μ, scaled so that the actually needed workload during execution fits into the available capacity. The scaled input is a virtual measure that is used to model the financial impact of downsizing. If the realized strategic project's workload P is smaller than or equal to the reserved capacity, the project can be exercised at its full scale and the scaled input I is equal to μ. In case P exceeds $\hat{M}$, we need to downsize the strategic project by a percentage $\hat{M}/P$ so that the scaled input $I = \mu \cdot \hat{M}/P$. The ratio $\hat{M}/P$ is related to the Cost Performance Index (CPI) encountered in the cost control literature on Earned Value Management (EVM) (cfr. Kerzner (1997)), where it represents the ratio of the budgeted cost for work performed (or 'earned value') over the actual cost of work performed (sometimes also simply called 'actual cost'). Within the EVM literature, the CPI is used to measure the performance of the project execution. A CPI < 1 indicates poor performance, and in our case leads to downsizing.

The ECV of a strategic project is modelled as follows.

$$y(\hat{M}; \alpha, \gamma, \mu, C) = \alpha I^{\gamma} C, \tag{1}$$

with

$$\begin{cases} I = \mu & \text{if } P \leq \hat{M}, \\ I = \mu \hat{M}/P & \text{if } P > \hat{M}, \end{cases} \tag{2}$$

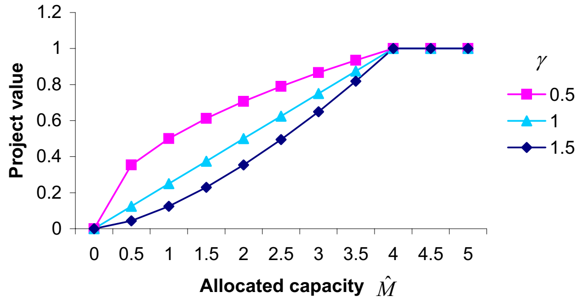


Figure 2: Project value for varying input $\hat{M}$, with γ equal to 0.5, 1 and 1.5 and deterministic workloads with $\mu = 4$.

with $\alpha > 0$ the project's production factor (cfr. the Cobb-Douglas functional form of production functions (Cobb & Douglas 1928)) and C a constant value. If we set $C = \mu^{1-\gamma}$, then the profit per workload $y(\hat{M}; \alpha, \gamma, \mu, C)/\mu$ of the project when it is not downsized is equal to $\frac{\alpha\mu^{\gamma}\mu^{1-\gamma}}{\mu}$, which corresponds to the production factor α.

The impact of downscaling on the project's overall value depends on the downsizeability parameter γ. When $0 < \gamma < 1$, the impact of a resource reduction is moderate. A plot of the value of a project with deterministic workloads against the input in manhours yields a concave curve; this corresponds with the lowest curve in Figure 2. This setting often holds for *incremental innovation projects* (Tushman et al. 1997), which aim to improve existing products and result in moderate productivity increases. When $\gamma = 1$, a linear relation between in- and output is implied. In case $\gamma > 1$, the effect of downsizing is detrimental to the project's revenue. This behavior is inherent in many *radical innovation projects*, which lead to radical improvements or completely new products. When successful, these projects lead to an upward productivity jump (Tushman et al. 1997). An example of the extreme case where $\gamma = \infty$ are the clinical testing trials during a drug development process, where downsizing is often not permitted due to strong governmental regulations.

3.2 Phase II: Allocate resources

Based on the properties of the project opportunities identified in Phase I, we select the projects to be executed in the next period and determine the allocated capacity. During its execution, a project can only employ the allotted resources. This is equivalent to the situation where all projects are processed *in parallel*, and should be distinguished from *sequential* planning, where unused resources can be shifted to later projects (De et al. (1993), Granot

& Zuckerman (1991)). As a direct consequence of this parallel planning policy, all projects planned within the same period finish at the end of the period in which they are executed. We assume that all revenues have been transformed into present values at the start of the period based on the cost of capital.

We establish the optimal selection by comparing the objective values of all feasible selections. A selection is feasible if the sum of the lower bounds of the projects in the selection is smaller than or equal to the available capacity M in a period. In a multi-project setting, an index k will be added to all parameters or variables that relate to a project k. Determining an optimal capacity allocation for a set of $\hat{N}$ projects corresponds to solving a non-linear program (NLP) of the following form:

$$\max z = \sum_{k=1,\ldots,\hat{N}} E[y_k(\hat{M}_k)] \tag{3}$$

subject to

$$\begin{array}{lll} \sum_{k=1,\ldots,\hat{N}} \hat{M}_k & \leq & M \\ \hat{M}_k & \leq & \overline{L}_k \quad (k = 1, \ldots, \hat{N}) \\ \hat{M}_k & \geq & \underline{L}_k \quad (k = 1, \ldots, \hat{N}). \end{array}$$

The allocated capacity $\hat{M}_k$ is bounded by a lower limit $\underline{L}_k$ and an upper limit $\overline{L}_k$. The presence of a lower downsizeability limit incorporates the fact that a project is not viable below a minimal resource input. The upper bound represents the resource input above which increases cease to produce additional profits.

3.3 Phase III: Downsize if required

When the project workloads are deterministic, the project's scale is a direct consequence of the allocated capacity and is known before the project's start. In such cases, the flexibility is maximal and we will mostly assume in this text that $\underline{L} = 0$ and $\overline{L} = \mu$. In the stochastic case, the actual required amount of resources (i.e. the realization of P) is only revealed during the execution of the project. If this exceeds the foreseen capacity, downsizing occurs. We assume that P follows a uniform distribution with bound values equal to $\mu(1 \pm \beta)$, with $0 < \beta < 1$; these values can also be interpreted as confidence limits for the estimated value μ. Since the need to downsize is only revealed during the execution of the project, the lower downsizeability bound $\underline{L}$ is set to $\mu(1 - \beta)$; the upper bound $\overline{L}$ is equal to $\mu(1 + \beta)$. The capacity that remains unused by strategic projects is assigned to utility projects.

4. The impact of downsizing on a strategic project's value

In this section, we assess the impact of downsizing on the project's value. For reasons of completeness, we set the lower bound $\underline{L}$ to 0, further in this text we will assume that $\underline{L} = \mu(1-\beta)$ in the stochastic case. The allocated amount is restricted by the available capacity M and by upper bounds $\mu(1+\beta)$ and μ for stochastic and deterministic workloads, respectively.

In the stochastic case, the strategic project needs to be downsized in case of a resource shortfall. This is incorporated in the reward function by reducing the strategic project's input. Two situations may occur: either the allocated capacity lies below the minimal realized workload $\mu(1-\beta)$ and downsizing is mandatory, or the allocated capacity is larger than or equal to $\hat{M}$, so that the need to downsize depends on the realization of P. We have

$$\begin{array}{lcl} E[y(\hat{M})|\hat{M} < \mu(1-\beta)] & = & E[\alpha(\mu\frac{\hat{M}}{P})^{\gamma}C] = \int_{\mu(1-\beta)}^{\mu(1+\beta)} f_P(p)\cdot\alpha(\mu\frac{\hat{M}}{p})^{\gamma}C\,dp, \\ E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)] & = & \int_{\mu(1-\beta)}^{\hat{M}} f_P(p)\cdot\alpha C\mu^{\gamma}\,dp + \int_{\hat{M}}^{\mu(1+\beta)} f_P(p)\cdot\alpha(\mu\frac{\hat{M}}{p})^{\gamma}C\,dp, \end{array} \tag{4}$$

with $f_P(p)$ the density function of random variable P. For deterministic workloads, $P = \mu$ and since $\hat{M} \leq \mu$, the project value is equal to

$$y(\hat{M}) = \alpha C\hat{M}^{\gamma}. \tag{5}$$

Proposition 1. *The function $E[y(\hat{M})]$ consists of two parts: $\hat{M} < \mu(1-\beta)$ and $\hat{M} \geq \mu(1-\beta)$. As long as $\hat{M} < \mu(1-\beta)$ (or $\hat{M} \leq \mu$ when workloads are deterministic), the allocated capacity $\hat{M}$ has increasing marginal returns when $\gamma > 1$, constant marginal returns when $\gamma = 1$ and decreasing marginal returns when $\gamma < 1$. Investments above $\mu(1-\beta)$ bring decreasing marginal returns for all values of γ.*

All proofs are relegated to the appendix. Proposition 1 states that the expected value of projects with stochastic workloads increases concavely between the minimal and the maximal realized workload. For projects with deterministic workloads an increase of the allocated capacity is associated with decreasing, linear or increasing marginal returns depending on the value of the downsizeability parameter. This result will be used in Section 5 to maximize the expected value of the single-period project portfolio.

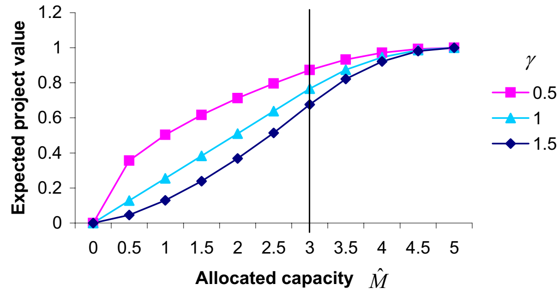


Figure 3: Expected value of a project with varying input and different values for γ equal to 0.5, 1 and 1.5 ($\mu = 4$ and $\beta = 0.25$).

Figure 2 showed the value of a project with a deterministic workload, different amounts of allocated capacity and for parameter γ equal to 0.5, 1 and 1.5 ($\mu = 4$). In Figure 3, the expected value of a project with a stochastic workload with varying resource input is given for $\beta = 0.25$. Capacity allocation above $\hat{M} = \mu(1-\beta) = 3$ leads to diminishing marginal returns for all values of the downsizeability parameter.

5. Selection and capacity allocation of multiple strategic projects

The sharing of resources makes the projects in a multi-project environment highly interdependent (Reiss 1996). In the extreme case where all projects are planned in parallel, resources allocated to a specific project can only be used by that project, while left-over resources can only be recovered by flexible utility projects. In Section 5.1 we study the problem for multiple projects with deterministic workloads and subsequently, Section 5.2 deals with stochastic workloads.

5.1 Multiple projects with deterministic workloads

When projects have deterministic resource requirements, then $E[y(\hat{M})] = \alpha C \hat{M}^{\gamma}$ and the objective function in Eq. (3) becomes

$$\max z = \sum_{k=1,\ldots,\hat{N}} \alpha_k C_k \hat{M}_k^{\gamma_k}, \tag{6}$$

with $\hat{M}_k \in [\underline{L}_k, \mu_k]$. This objective function is no longer guaranteed to be concave when their exists a project k for which $\gamma_k > 1$. An optimal solution of this NLP needs to satisfy the Kuhn-Tucker conditions (Bertsekas 1995). When $\underline{L}_k = 0$ for all $k = 1, \ldots, N$, project selection is implicit from the solution of the NLP, since projects with $\hat{M}_k = 0$ are no longer selected. If we cannot provide all projects with their maximal amount of capacity then Proposition 2 holds.

Proposition 2. *At most three values need to be considered for the resource allocation of every project k in the selection: $\underline{L}_k, \mu_k$ and $\left(\frac{\lambda}{\alpha_k C_k \gamma_k}\right)^{\frac{1}{\gamma_k - 1}}$, with $\lambda > 0$. At most one of the projects in the selection with $\gamma \geq 1$ will receive an intermediate amount (i.e. an amount strictly between the lower and upper bound) of capacity.*

In other words, the allocation of a project for which downsizing has a large ($\gamma \geq 1$) impact generally becomes an all-or-nothing decision and only when $\gamma < 1$, downsizing is commonly applied. This result is related to the findings of Loch & Kavadias (2002), who allocate the total capacity to a single program or project if all projects have increasing returns. In case the set of projects have decreasing returns, the budget is split according to the projects' marginal benefits.

5.2 Multiple projects with stochastic workloads

Since for stochastic workloads, the need to downsize is only revealed during the execution of the project, the lower downsizeability bound $\underline{L}_k$ is set to $\mu_k(1-\beta_k)$ for every project k in the stochastic setting, so that $\mu_k(1-\beta_k) \leq \hat{M}_k \leq \mu_k(1+\beta_k)$. Since we observed in Proposition 1 that investments above $\mu_k(1-\beta_k)$ bring decreasing marginal returns, the objective function z in Eq. (3) is a sum of concave functions. The resources are therefore distributed optimally if the following conditions hold (Bertsekas (1995), p. 432).

$$\begin{array}{rcll} \frac{\partial E[y_k(\hat{M}_k)]}{\partial \hat{M}_k} & = & \lambda & \text{if } \mu_k(1-\beta_k) < \hat{M}_k < \mu_k(1+\beta_k), \\ \frac{\partial E[y_k(\hat{M}_k)]}{\partial \hat{M}_k} & \leq & \lambda & \text{if } \hat{M}_k = \mu_k(1-\beta_k), \\ \frac{\partial E[y_k(\hat{M}_k)]}{\partial \hat{M}_k} & \geq & \lambda & \text{if } \hat{M}_k = \mu_k(1+\beta_k), \end{array} \tag{7}$$

with $\lambda \geq 0$ the Lagrange multiplier and $\lambda(\sum_{k=1,\ldots,\hat{N}} \hat{M}_k - M) = 0$. These conditions show that projects can receive any amount of capacity between the upper and lower capacity bounds. The influence of the project parameters that we observe is as follows. The optimal selection is obtained by enumerating all feasible selections and comparing the objective values corresponding with an optimal solution obtained from Eq. (7).

Proposition 3. *If two projects have the same parameter values except for C, respectively α, then the project with the higher value for C, respectively α, will never receive less capacity than the other project.*

Proposition 3 indicates that a project that has a larger production factor or constant value than a similar project is more precious and must therefore be appointed at least as much capacity as the similar project.

The following proposition holds for any two projects for which the available capacity is larger than the higher lower capacity bound.

Proposition 4. *If two projects have the same parameter values except for μ, then the project with the higher value for μ receives the most capacity.*

When two projects are alike except for their average workload, Proposition 4 exhibits that the project with the higher workload receives the most capacity under the restriction that sufficient capacity is available to satisfy its lower capacity bound.

Proposition 5. *If two projects have the same parameter values except for β ($\beta_1 > \beta_2$) and their optimal capacities are $\hat{M}_1$ and $\hat{M}_2$ respectively, then it follows that:*

(1) *if $\mu_2(1-\beta_2) \le \hat{M}_k < M_2^*$ for $k \in \{1,2\}$ then $\hat{M}_1 \le \hat{M}_2$;*

(2) *if $M_2^* \le \hat{M}_k < M_1^*$ for $k \in \{1,2\}$ then the highest allocation can go either way;*

(3) *if $M_1^* \le \hat{M}_k$ for $k \in \{1,2\}$ then $\hat{M}_1 \ge \hat{M}_2$;*

with

$$\begin{aligned} M_k^* &= \mu_k(1+\beta_k)\exp\left(\tfrac{-\beta_k}{1+\beta_k}\right) && \text{if } \gamma_k = 1, \\ &= \mu_k(1+\beta_k)\left(\tfrac{1+\beta_k}{1+\beta_k\gamma_k}\right)^{1/(\gamma_k-1)} && \text{if } \gamma_k \neq 1. \end{aligned}$$

Proposition 5 describes the relation between the optimal capacity of two similar projects with different coefficients for the uniform distribution. When both projects obtain little capacity (1) and resources are scarce, the project with the highest workload uncertainty receives the least capacity. In this case, the probability of downsizing is high for both projects, so that it is preferable to invest resources in the more certain project. In case (3), when both projects are appointed more than the average workload ($\mu_k, k \in \{1,2\}$, is clearly larger than M_k^* and $M_2^* \le M_1^*$), the project with the lowest workload uncertainty receives the least capacity. In this case, the probability of downsizing is low for both projects, so additional capacity goes to the project with low workload variability. For the remaining cases no conclusions can be drawn.

6. Portfolio behavior

To solve the multi-period project selection and capacity allocation problem, we are interested in the behavior of a portfolio. More specifically, we demonstrate through simulation experiments and statistical analysis that the expected value of a portfolio of more than 10 projects decreases linearly when we moderately (by at most two times the average project size) reduce the portfolio capacity.

During multiple (50) simulation runs we generate sets of random projects for which the optimal selection is computed. From the optimal capacity allocations, the expected portfolio values are obtained, which serve as inputs for the statistical analysis. An overview of the factors varied during the simulation experiments is given in Table 1. The capacity M available for allocation is $5lN$, with $l \in]0, 1]$ the load parameter, indicating the ratio of the joint average workload $5N$ of the N projects and the available capacity M. We subsequently reduce the capacity M by one and two times the average project workload.

Table 1: Factors of the simulation experiment.

factor	name	values
α	production factor	$U(0, 10)$
C	constant value	$\mu^{1-\gamma}$
μ	average workload	$U(0, 10)$
β	parameter of the workload distribution	$U(0, 1)$
N	number of projects available	10, 20
l	load parameter	0.6, 0.8

Based on the simulation results for portfolios of projects with deterministic and stochastic workloads we test the following hypothesis.

Hypothesis 1. *The expectation of the value Y of a portfolio obtained by the model decreases linearly when the portfolio capacity M is moderately (by maximal two times the average project size) reduced.*

To test Hypothesis 1 we run a pooled general linear regression with $E[Y]$ as the dependent variable and one explanatory variable M; we add l, the load parameter, as a control variable. The pools follow from N, the number of available projects. The regression is performed with cross-section weights to counter the heteroskedasticity resulting from the different pools; diagnostic tests reveal no other issues. The coefficient of determination R^2 for the model with stochastic workloads is 85% and amounts to 82% for the model with deterministic loads.

The estimates for the influence of the portfolio capacity M for both pools are all significant at the ($p < 0.001$)-level.

We find support for Hypothesis 1, and continue to model the behavior of a portfolio as a project with linear ($\gamma = 1$) downsizeability, with $C = 1$ and $\mu = M$. This behavior is justified as long as only moderate reductions of the project size are performed. The production factor α is fixed and corresponds to the ROI of the whole portfolio, which is denoted as $\alpha = \alpha_{ROI}$.

7. Multi-period project selection and capacity allocation

Traditionally, the multi-period selection problem for one- or multi-stage projects is solved by collapsing it into a one-period problem in which future opportunities and uncertainties are captured by a decision tree. These decision trees treat projects independently from each other (see e.g. Hess (1993), Kavadias & Loch (2004)) and implicitly assume that sufficient resources can always be found. In this section, we investigate how the value of multi-stage projects is affected when resources are limited and under what circumstances we can treat projects independently from each other.

We allow for projects to advance in stages or subprojects. Between these stages a complete order is imposed and some of the stages are only reached with a certain probability (cfr. stage-gating (Cooper et al. 1998)). We assume that per project at most one project stage can be selected in every period and that, if the project is successful, the project revenue is reaped at the end of the final stage. For ease of analysis we assume that downsizing a multi-stage project can only occur in the final stage and that the revenue of the project will depend on the scale of that stage. The possibility of downscaling all project stages is discussed in Section 8.1. The first stage of a fictitious two-stage project may for instance comprise the development of a new fabric for high-tech swimsuits (SP1), while the second subproject may deal with the restyling of the bathing suits (SP2).

In this section, we consider a multi-period setting where a new allocation decision is made at the start of each period. The multi-period problem aims to determine the optimal portfolio at the start of the first period, assuming that optimal decisions will be made in all subsequent periods t, $t = 2, 3, \ldots, T$. To determine the optimal decision at time zero, we move backward, starting from the optimal selection and capacity-allocation decision in the final period T; a visualization of this process is provided in Figure 4. In Section 6, we advanced that a

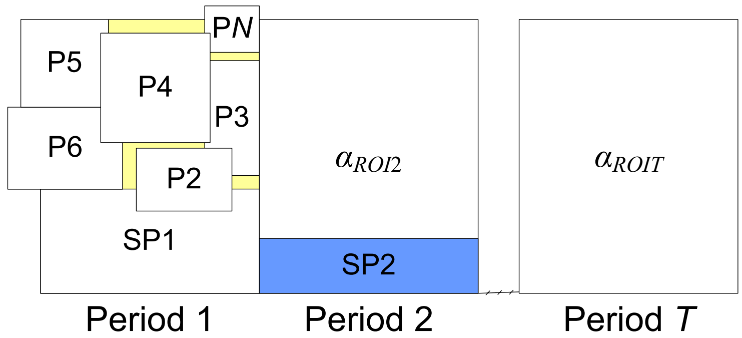


Figure 4: The multi-period selection and capacity-allocation process for a two-stage project consisting of SP1 and SP2.

sufficiently large portfolio behaves as a project with linear downsizeability ($\gamma = 1$), $C = 1$, $\alpha = \alpha_{ROI}$ and $\mu = M$ if only moderate reductions of the project size are allowed. We now assume that the return on investment for all subsequent periods $t = 2, 3, \ldots, T$ can be estimated and is equal to α_{ROIt}. Since the capacity allocation of the final-stage subproject (in casu SP2 in Figure 4) is the key driver of the value of multi-stage projects, this outcome contains essential information for the decision-making process in all previous periods. The decision problem in period 1 then corresponds to the single-period problem described in Section 5.

In Section 7.1 we determine the expected value of a (sub)project for an estimated return on investment for periods 2 to T during the multi-period process. In the pharmaceutical industry, for example, the average return on investment is said to be much higher than the weighted average cost of capital (WACC) (Brealey & Myers 2003) and typically lies between 20% and 30% (Loch & Bode-Greuel 2001). When projects may fail, continuation in the following periods is uncertain. In addition, new project opportunities may arise that have unknown profits and resource requirements. Both events have an effect on the resource availability during subsequent periods (cfr. Ding & Eliashberg (2002)). Under these circumstances, the return on investment is the result of these different sources of uncertainty and becomes scenario-dependent; this case is the subject of Section 7.2.

7.1 Project value for an estimated return on investment

We now derive the value of a project that is either a single-stage project or a subproject that is the final stage of a larger project while the ROI of its final execution period is estimated to be α_{ROIt}, with $t \in \{2, 3, \ldots, T\}$. In this section, we determine the resources to reserve and the project value at the start of period t, which are required to solve the single-period problem in the first period. We consider decision period $t \in \{2, 3, \ldots, T\}$ which has an available periodic capacity M_t. Since we always study period t in this section, we drop the index t for ease of notation. The different cases that may occur during the decision period are the following:

Case (a) $M < \underline{L}$: the (sub)project cannot be accepted in the decision period due to a lack of available resources. The value of $E[y(\hat{M})|\hat{M} \leq M]$ is therefore 0.

Case (b) $M \geq \underline{L}$: the amount of allocated capacity will depend on the project properties. The obtained $\hat{M}$ allows us to determine the value of the (sub)project.

The derivation of the expected value of the (sub)projects in case (b) for deterministic and for stochastic workloads is the topic of Section 7.1.1 and Section 7.1.2 respectively. One can easily incorporate the possibility of failure or stage-gating into these calculations, by multiplying the (sub)project value by the success rate at the gate.

7.1.1 Multi-period project selection with deterministic workloads

We first study the value of the project with certain workloads ($\beta = 0$) and $\underline{L} = 0$. Later on in this section, we raise the lower downsizeability bound to $\underline{L} > 0$.

Proposition 6. *The optimal values for $\hat{M}$ for a project with deterministic workloads can be found in Table 2; we distinguish between the situation where γ is smaller than, larger than or equal to 1. In addition, the allocation depends on whether $\mu \leq M$ or $\mu > M$.*

Table 2: Optimal resource allocation for deterministic workloads.

<table>
<tr><td></td><td>$\mu \leq M$</td><td>$\mu > M$</td></tr>
<tr><td rowspan="2">$\gamma < 1$</td><td colspan="2">$m_1 = \left(\frac{\alpha_{ROI}}{\gamma \alpha C}\right)^{1/(\gamma-1)} \Rightarrow$</td></tr>
<tr><td>If $m_1 \geq \mu \Rightarrow \hat{M} = \mu$
else $\hat{M} = m_1$</td><td>If $m_1 \geq M \Rightarrow \hat{M} = M$
else $\hat{M} = m_1$</td></tr>
<tr><td rowspan="2">$\gamma = 1$</td><td colspan="2">$\alpha_{ROI} \geq \alpha C \Rightarrow \hat{M} = 0$
$\alpha_{ROI} < \alpha C \Rightarrow$</td></tr>
<tr><td>$\hat{M} = \mu$</td><td>$\hat{M} = M$</td></tr>
<tr><td rowspan="2">$\gamma > 1$</td><td colspan="2">$m_2 = \left(\frac{\alpha_{ROI}}{\alpha C}\right)^{1/(\gamma-1)} \Rightarrow$</td></tr>
<tr><td>If $m_2 \leq \mu \Rightarrow \hat{M} = \mu$
else $\hat{M} = 0$</td><td>If $m_2 \leq M \Rightarrow \hat{M} = M$
else $\hat{M} = 0$</td></tr>
</table>

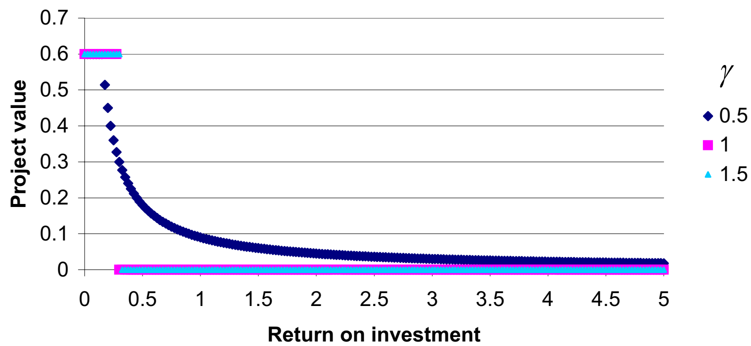


Figure 5: Value of projects with varying return on investment for different values of γ and $\mu \leq M$.

When the optimal value for $\hat{M}$ is known, the project value is simply obtained as $\alpha C \hat{M}^\gamma$. From Proposition 6, we derive that in case of linear or increasing returns to scale ($\gamma \geq 1$) the project takes on the extreme value 0 or $\alpha C \hat{M}^\gamma$, with $\hat{M} = \min(\mu, M)$. This implies that if $\gamma \geq 1$ the portfolio ROI constitutes a hurdle rate on the average project ROI, below which the project becomes worthless. A hurdle rate is the minimum rate of return that must be met for a company to undertake a particular project. The selection thus becomes an all-or-nothing decision. When downsizing is associated with diminishing returns ($\gamma < 1$), the project may take on intermediate values depending on m_1 as specified in Proposition 6; this leads to $y(m_1) = \alpha C \left(\frac{\alpha_{ROI}}{\alpha C \gamma}\right)^{\frac{\gamma}{\gamma-1}}$. When the lower feasibility limit is raised ($\underline{L} > 0$), the allocated capacity is zero if $\gamma < 1$ and $m_1 < \underline{L}$. Obviously, a restriction of the downsizing flexibility has a negative effect on the project value. A visualization of the project values related to the return on investment (α_{ROI}) of the portfolio for $\gamma \in \{0.5; 1; 1.5\}$ and $\mu \leq M$ is given in Figure 5.

Proposition 7. *When $\gamma < 1$, the expected value of the (sub)project decreases with the return on investment (α_{ROI}), this decrease is convex.*

In Figure 5 we observe that when the rate of return (α_{ROI}) is sufficiently low, the appointed capacity is maximal and the project value equals the independent project value (i.e. the project value when $\alpha_{ROI} = 0$). The received capacity is in this case only restricted by the total amount of available resources M.

Proposition 8. *When the return on investment (α_{ROI}) lies below $\alpha C \gamma \mu^{\gamma-1}$, αC and $\alpha C \mu^{\gamma-1}$ for $\gamma < 1$, $\gamma = 1$ and $\gamma > 1$, respectively, projects can be treated independently from each other.*

From Proposition 7 and Proposition 8, we conclude that the impact of scarce resources on the value of a project cannot be fully captured by the common financial practice of adapting the discount rate with the estimated return on investment.

7.1.2 Multi-period project selection with stochastic workloads

The optimal capacity allocation for projects in the stochastic setting is characterized in Proposition 9. The expected value $E[y(\hat{M})]$ of a (sub)project with stochastic workloads is derived in the appendix (Proof Proposition 1).

Proposition 9. *The optimal values for $\hat{M}$ depend on whether $\gamma = 1$ and if $\mu(1+\beta) \leq M$ or $\mu(1-\beta) \leq M < \mu(1+\beta)$; they are given in Table 3.*

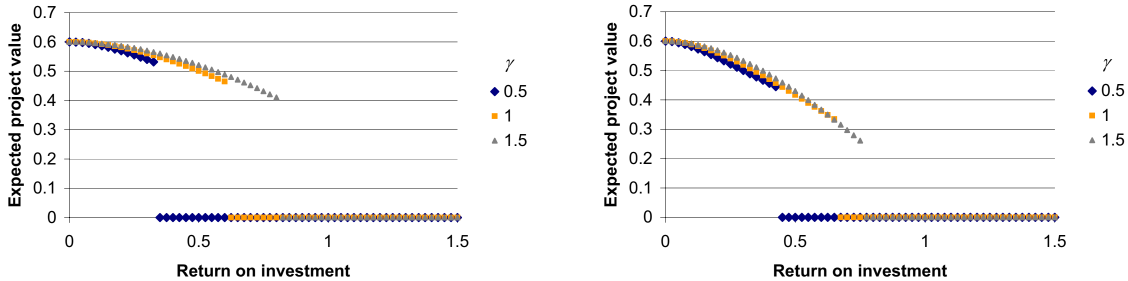
Table 3: Optimal resource allocation for stochastic workloads.

<table>
<tr><td></td><td>$\mu(1+\beta) \leq M$</td><td>$\mu(1-\beta) \leq M < \mu(1+\beta)$</td></tr>
<tr><td rowspan="2">$\gamma \neq 1$</td><td colspan="2">$m_1 = (1+\beta)\left(\frac{\alpha_{ROI} 2\beta(1-\gamma)}{C\alpha\gamma} + \mu^{\gamma-1}\right)^{\frac{1}{\gamma-1}} \Rightarrow$</td></tr>
<tr><td>$m_1 \geq \mu(1+\beta) \Rightarrow \hat{M} = \mu(1+\beta)$
$m_1 < \mu(1-\beta) \Rightarrow \hat{M} = 0$
else $\hat{M} = m_1$</td><td>$m_1 \geq M \Rightarrow \hat{M} = M$
$m_1 < \mu(1-\beta) \Rightarrow \hat{M} = 0$
else $\hat{M} = m_1$</td></tr>
<tr><td rowspan="2">$\gamma = 1$</td><td colspan="2">$m_2 = \mu(1+\beta)\exp\left(-\frac{\alpha_{ROI} 2\beta}{\alpha C}\right) \Rightarrow$</td></tr>
<tr><td>$m_2 \geq \mu(1+\beta) \Rightarrow \hat{M} = \mu(1+\beta)$
$m_2 < \mu(1-\beta) \Rightarrow \hat{M} = 0$
else $\hat{M} = m_2$</td><td>$m_2 \geq M \Rightarrow \hat{M} = M$
$m_2 < \mu(1-\beta) \Rightarrow \hat{M} = 0$
else $\hat{M} = m_2$</td></tr>
</table>

Figure 6 shows how the project values vary with the return on investment (α_{ROI}) of the portfolio for $\gamma \in \{0.5; 1; 1.5\}$, $\beta \in \{0.25; 0.5\}$ and $\mu(1+\beta) \leq M$. Although the values of the projects with $\gamma = 1.5$ will be affected the most by a downsize, their expected project values are higher than those of the projects with smaller downsizeability parameters. The reason for this is that the former receive more capacity to limit the risk of downsizing and as a result exhibit a higher expected value.

Proposition 10. *The expected value of the (sub)project decreases concavely with the ROI of the portfolio: for $\gamma < 1$, the decrease becomes convex when $\alpha_{ROI} > \frac{\alpha C}{2\beta}\gamma\mu^{\gamma-1}$. For $\gamma = 1$, the decrease becomes convex when $\alpha_{ROI} > \frac{\alpha C}{2\beta}$.*

Proposition 10 states that the expected value of a (sub)project decreases concavely with the ROI of the portfolio. For projects with decreasing returns to scale the decrease becomes



(a) $\beta = 0.25$ (b) $\beta = 0.5$

Figure 6: Expected project values with return on investment for different values of γ and $\mu(1+\beta) \leq M$.

convex when the ROI is equal to the ratio of the marginal project return and two times the parameter of the workload distribution. For projects with constant returns to scale the decrease becomes convex once the portfolio ROI is higher than the ratio of the average project return and two times the parameter of the workload distribution.

Proposition 11. *Projects with stochastic workloads can only be treated independently if* $\alpha_{ROI} = 0$.

The expected project value is only equal to the independent project value if we do not take the risk of downsizing and allocate the maximum amount of capacity to the project. Proposition 11 reveals that only if the portfolio return is equal to zero, we do not undergo the risk of downsizing selected projects. From Proposition 10 and Proposition 11, we again conclude that the impact of resource scarcity on the expected value of a project cannot be correctly incorporated by merely adapting the discount rate.

7.2 Project value with a scenario-dependent return on investment

Uncertainties such as a project's success or failure and the arrival of new project opportunities have an impact on the return on investment of the project portfolio in future periods. This is illustrated by a small example consisting of four projects that become available in the following period. The first project (P1) is a new project opportunity that has a probability of arrival $Prob$ equal to 0.8. The second project (P2) is the second-stage subproject of a two-stage project, the success rate of the first-stage subproject is 0.5, which is also the subproject's probability of arrival. Projects 3 and 4 are certainly available during the following selection period. All project characteristics are given in Table 4.

Table 4: Project charactistics.

Property	New opportunity (P1)	Subproject (P2)	Project (P3)	Project (P4)
γ	1	0.5	1.2	1
α	0.3	0.2	0.3	0.1
C	2	0.7	1	0.4
μ	0.5	2	1	2.5
β	0.2	0.4	0.3	0.4
$Prob$	0.8	0.5	1	1

From the probabilities of arrival of each (sub)project a number of possible sets of candidate projects can be obtained. Based on the model in Eq. (7), we derive the optimal portfolio composition and the capacity allocation for every project; the results for an available amount of capacity equal to 4 are gathered in Table 5. Pr represents the probability of occurrence of the scenario. The average portfolio return on investment of the following period is represented by $\overline{\alpha}_{ROI}$ and equal to 0.17 for our small example.

Table 5: Return on investment and probability of occurrence of the optimal portfolios.

Available set	$\hat{M}_{P1}$	$\hat{M}_{P2}$	$\hat{M}_{P3}$	$\hat{M}_{P4}$	Pr	α_{ROI}
$\{P1, P2, P3, P4\}$	0.6	2.1	1.3	0	0.4	0.20
$\{P2, P3, P4\}$	0	1.2	1.2	1.6	0.1	0.13
$\{P1, P3, P4\}$	0.6	0	1.2	2.2	0.4	0.17
$\{P3, P4\}$	0	0	1.2	2.8	0.1	0.10

The actual ROI is the result of the decisions made during the selection process in each period. For multi-stage projects this implies that after an initial investment, management can gather more information on the specific scenario that unfolds, and adapt its course of action accordingly. Under these circumstances, a traditional NPV analysis of the project value no longer suffices and a real options approach (ROA) is required to determine the true (sub)project value (see e.g. Dixit & Pindyck (1994), Trigeorgis (1997)):

$$ROA = E[y(\hat{M})|\alpha_{ROI}]. \tag{8}$$

In this equation, the allocated capacity $\hat{M}$ varies with the estimated ROI of the portfolio; this is motivated by the findings of Section 7.1. When calculating the ROA value of a single project, we implicitly assume that the selection of a single project does not affect the ROI of the whole portfolio. This assumption seems reasonable for real-size portfolios.

The expected value of a project for the distribution of the ROI given in Table 5, can be calculated as $E[y(\hat{M})|\alpha_{ROI}] = 0.4 \cdot E[y(\hat{M})|\alpha_{ROI} = 0.20] + 0.1 \cdot E[y(\hat{M})|\alpha_{ROI} = 0.13] +$

$0.4 \cdot E[y(\hat{M})|\alpha_{ROI} = 0.17] + 0.1 \cdot E[y(\hat{M})|\alpha_{ROI} = 0.10]$. As a result, the expected value of a (sub)project with parameter values $\gamma = 2$, $\alpha = 0.18$, $C = 0.5$, $\mu = 2$ and $\beta = 0.3$ at the start of its execution period corresponds to $E[y(\hat{M})|\alpha_{ROI}] = 0.4 \cdot 0.27 + 0.1 \cdot 0.32 + 0.4 \cdot 0.30 + 0.1 \cdot 0.34 = 0.29$. This value differs from the NPV value, for which the capacity amount reserved for the project is assumed to be fixed depending on the average ROI: $E[y(\hat{M})|\overline{\alpha}_{ROI} = 0.17] = 0.3$. The expected project value calculated independently of the resource availability becomes $E[y(\hat{M})|\alpha_{ROI} = 0] = 0.36$. This illustration shows that neglecting the limited availability of resources results in an overestimation of the expected project value.

8. Model extensions

Some model extensions are briefly discussed in this section. We consider the downsizing of multiple project stages in Section 8.1 and subsequently incorporate the presence of multiple resources in Section 8.2.

8.1 Downsizing of multiple project stages

When downsizing can occur in multiple substages of a project and when this affects the revenue at execution time, the optimal capacity allocations need to be calculated simultaneously for all periods. We consider a T-period problem and assume that the portfolio in every period $t = 1, 2, \ldots, T$ can be modeled as a single project with an estimated return on investment α_{ROIt}, as we have done previously in Section 7 for $t > 1$. The subproject characteristics carry an index $j \in \{1, 2, \ldots, T\}$ denoting the subproject they belong to; for this example the index value coincides with the period in which they are executable. The ECV of the strategic project is obtained from Eq. (1) in which we replace $\hat{M}$ with the allocated capacity $\hat{M}_1$ in the first downsizeable stage. The allocated amounts of capacity in later periods are obtained through rescaling, so that $\hat{M}_t = f_t(\hat{M}_1)$. In the stochastic case $f_t(\hat{M}_1) = (\hat{M}_1 - \mu_1(1 - \beta_1))\frac{\beta_t \mu_t}{\beta_1 \mu_1} + \mu_t(1 - \beta_t)$ with $t \in \{2, 3, \ldots, T\}$. This implies that when the minimal (maximal) amount of capacity is allocated to subproject 1, subproject t ($t \in \{2, 3, \ldots, T\}$) also receives a minimal (maximal) allocation. This relies on the assumption that the realizations of P_t ($t \in \{2, 3, \ldots, T\}$) deviate from μ_t with the same proportion as P_1 deviates from μ_1 in the first stage. The scale for downsizing is thus determined in the first stage and remains unchanged throughout the whole project execution. For deterministic workloads $f_t(\hat{M}_1) = \hat{M}_t = \hat{M}_1 \mu_t / \mu_1$ with $t \in \{2, 3, \ldots, T\}$, the scale factor $\hat{M}_1/\mu_1$ is determined prior to the project execution.

The project scale that maximizes the expected value of the portfolio is determined by an NLP with linear constraints:

$$\max z = E[y(\hat{M}_T)] - \sum_{t=1,\dots,T} \alpha_{ROIt}\hat{M}_t$$

subject to

$$\begin{array}{rcll} \hat{M}_t & = & f_t(\hat{M}_1) & (t = 2, \dots, T), \\ \hat{M}_t & \leq & \overline{L}_t & (t = 1, \dots, T), \\ \hat{M}_t & \geq & \underline{L}_t & (t = 1, \dots, T). \end{array}$$

For stochastic workloads, it follows from Proposition 1 that the first part of the objective function is a concave function and that the second part is a sum of linear functions. The solution can therefore be derived analogously to the solution in Eq. (7). In the deterministic case, the shape of the objective function depends on the value of γ, so that the optimal solution must be obtained from the Kuhn-Tucker conditions. The NLP trades off the project's revenue for different capacity allocations and the capacity's opportunity cost incorporated by the return on investment of the portfolio in every period. In case $z \leq 0$, no capacity should be allocated to the project.

8.2 Multiple resources

We can extend the model for R different resource types by considering different functions such as manager, project engineer and technician. The ECV of a strategic project now becomes:

$$y(\hat{M}_1, \hat{M}_2, \dots, \hat{M}_R; \alpha, \gamma_1, \gamma_2, \dots, \gamma_R, \mu, C) = \alpha C \prod_{j=1}^{R} I_j{}^{\gamma_j},$$

with

$$\begin{cases} I_j = \mu_j & \text{if } P_j \leq \hat{M}_j, \\ I_j = \mu_j \hat{M}_j / P_j & \text{if } P_j > \hat{M}_j. \end{cases}$$

Every resource j has its own downsizeability factor γ_j. Based on this more general payoff function, an extension of our model to multiple resources can be undertaken, but would not lead to additional insights.

9. Conclusions

In this paper, we have determined the optimal capacity allocation for both single and multi-period project portfolios, while taking into account the possibility of downsizing and limited resource availability. We have found that when the project workloads are known, the optimal resource allocation for the single-period optimization problem is usually an all-or-nothing decision for projects with constant or increasing returns to scale. For decreasing returns to scale and also in case of stochastic workloads, the optimal allocation depends on the marginal benefits of the projects.

In order to solve the multi-period problem, we have modeled the behavior of the portfolio in subsequent periods as a single project for which the return on investment (ROI) can be estimated. When this setting is unappropriate and the ROI is scenario-dependent, precommitment to any action is not necessarily optimal and one needs to turn to real option analysis to correctly incorporate the value of managerial flexibility in the portfolio appreciation.

For given estimates of the portfolio ROI, we have established the impact of resource scarcity on the value of downsizeable projects. We find that limited resource availability can be neglected only when the ROI of the portfolio is sufficiently low. When a project has known workloads and linear or increasing returns, the portfolio ROI constitutes a hurdle rate on the average project ROI, below which the project becomes worthless; project selection thus corresponds to an all-or-nothing decision. When returns are decreasing, on the other hand, the project value goes down convexly with the ROI of the portfolio. When workloads are not perfectly known from the start, however, the expected project value generally decreases concavely with the portfolio's ROI, and again becomes worthless once the portfolio ROI exceeds a specific measure of project profitability. These findings contrast strongly with the common practice in Finance to incorporate the impact of scarce resources simply by adapting the discount rate.

Appendix: proofs

Proof (Proposition 1): From Eq. (4), we derive the expected project values for the stochastic case. These take on a different form if γ is equal to 1 or not and if $\hat{M}$ is smaller than, or equal to or larger than $\mu(1-\beta)$.

$$
\begin{array}{l}
\text{If } \gamma \neq 1 \Rightarrow \\
\left\{
\begin{array}{lcl}
E[y(\hat{M})|\hat{M} < \mu(1-\beta)] & = & \frac{\alpha C \hat{M}^{\gamma}}{2\beta(1-\gamma)}\Big((1+\beta)^{1-\gamma} - (1-\beta)^{1-\gamma}\Big), \\
E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)] & = & \frac{\alpha C}{2\beta}\Big(\hat{M}\mu^{\gamma-1} - \mu^{\gamma}(1-\beta) + \frac{1}{1-\gamma}\Big(\hat{M}^{\gamma}(1+\beta)^{1-\gamma} - \hat{M}\mu^{\gamma-1}\Big)\Big);
\end{array}
\right. \\
\text{if } \gamma = 1 \Rightarrow \\
\left\{
\begin{array}{lcl}
E[y(\hat{M})|\hat{M} < \mu(1-\beta)] & = & \frac{\alpha C \hat{M}}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right), \\
E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)] & = & \frac{\alpha C}{2\beta}\Big(\hat{M} - \mu(1-\beta) + \hat{M}\ln\left(\frac{\mu(1+\beta)}{\hat{M}}\right)\Big).
\end{array}
\right.
\end{array}
\tag{9}
$$

From these values we obtain partial derivatives of the expected project values.

$$
\begin{array}{l}
\text{If } \gamma \neq 1 \Rightarrow \\
\left\{
\begin{array}{lcl}
\frac{\partial E[y(\hat{M})|\hat{M} < \mu(1-\beta)]}{\partial \hat{M}} & = & \frac{\alpha C \gamma \hat{M}^{\gamma-1}}{2\beta(1-\gamma)}\Big((1+\beta)^{1-\gamma} - (1-\beta)^{1-\gamma}\Big), \\
\frac{\partial E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]}{\partial \hat{M}} & = & \frac{\alpha C \gamma}{2\beta(1-\gamma)}\Big(\hat{M}^{\gamma-1}(1+\beta)^{1-\gamma} - \mu^{\gamma-1}\Big);
\end{array}
\right. \\
\text{if } \gamma = 1 \Rightarrow \\
\left\{
\begin{array}{lcl}
\frac{\partial E[y(\hat{M})|\hat{M} < \mu(1-\beta)]}{\partial \hat{M}} & = & \frac{\alpha C}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right), \\
\frac{\partial E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]}{\partial \hat{M}} & = & \frac{\alpha C}{2\beta}\ln\left(\frac{\mu(1+\beta)}{\hat{M}}\right).
\end{array}
\right.
\end{array}
\tag{10}
$$

From these functions, we derive the second-order partial derivatives.

$$
\begin{array}{l}
\text{If } \gamma \neq 1 \Rightarrow \\
\left\{
\begin{array}{lcl}
\frac{\partial^2 E[y(\hat{M})|\hat{M} < \mu(1-\beta)]}{\partial^2 \hat{M}} & = & -\frac{\alpha \gamma C \hat{M}^{\gamma-2}}{2\beta}\Big((1+\beta)^{1-\gamma} - (1-\beta)^{1-\gamma}\Big), \\
 & < & 0, \text{ if } \gamma < 1, \\
 & > & 0, \text{ if } \gamma > 1, \\
\frac{\partial^2 E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]}{\partial^2 \hat{M}} & = & -\frac{\alpha C \gamma}{2\beta}\hat{M}^{\gamma-2}(1+\beta)^{(1-\gamma)} < 0;
\end{array}
\right. \\
\text{if } \gamma = 1 \Rightarrow \\
\left\{
\begin{array}{lcl}
\frac{\partial^2 E[y(\hat{M})|\hat{M} < \mu(1-\beta)]}{\partial^2 \hat{M}} & = & 0, \\
\frac{\partial^2 E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]}{\partial^2 \hat{M}} & = & -\frac{\alpha C}{2\beta} < 0.
\end{array}
\right.
\end{array}
\tag{11}
$$

From Eq. (5), we derive the first-order partial derivative for the deterministic case.

$$
\begin{array}{l}
\text{If } \gamma \neq 1 \Rightarrow \frac{\partial y(\hat{M})}{\partial \hat{M}} = \alpha C \gamma \hat{M}^{\gamma-1}, \\
\text{if } \gamma = 1 \Rightarrow \frac{\partial y(\hat{M})}{\partial \hat{M}} = \alpha C.
\end{array}
$$

The second-order partial derivative for the deterministic case are the following.

$$
\begin{array}{l}
\text{If } \gamma \neq 1 \Rightarrow \frac{\partial^2 y(\hat{M})}{\partial^2 \hat{M}} = \alpha C \gamma(\gamma - 1)\hat{M}^{\gamma-2}, \\
\text{if } \gamma = 1 \Rightarrow \frac{\partial^2 y(\hat{M})}{\partial^2 \hat{M}} = 0.
\end{array}
$$

If $\gamma > 1$ the second-order partial derivative is larger than zero and if $\gamma < 1$ the derivative is negative. $\square$

Proof (Proposition 2): Maximizing the Lagrangean $\mathcal{L}(\hat{M}_1, \hat{M}_2, \ldots, \hat{M}_{\hat{N}}) = \sum_k \alpha_k C_k \hat{M}_k^{\gamma_k} + \lambda(M - \sum_k \hat{M}_k) + \sum_k \theta_k(\mu_k - \hat{M}_k) + \sum_k \omega_k(\hat{M}_k - \underline{L}_k)$, with λ, θ_k and ω_k the Lagrange multipliers (≥ 0), results in the following Kuhn-Tucker conditions

$$\begin{array}{lcl} \alpha_k C_k \gamma_k \hat{M}_k^{\gamma_k - 1} - \lambda - \theta_k + \omega_k & = & 0 \ \forall k, \\ \lambda(M - \sum_k \hat{M}_k) & = & 0, \\ \theta_k(\mu_k - \hat{M}_k) & = & 0 \ \forall k, \\ \omega_k(\hat{M}_k - \underline{L}_k) & = & 0 \ \forall k. \end{array}$$

For every project k with $\mu_k \neq \underline{L}_k$, it follows that if $\theta_k \neq 0 \Rightarrow \omega_k = 0$ and if $\omega_k \neq 0 \Rightarrow \theta_k = 0$. The third possibility is that $\theta_k = \omega_k = 0$, so that the three possible values for $\hat{M}_k$ are $\underline{L}_k, \mu_k$ and $\left(\frac{\lambda}{\alpha_k C_k \gamma_k}\right)^{\frac{1}{\gamma_k - 1}}$. This third value can only be obtained if all capacity is allocated ($M = \sum_k \hat{M}_k$). We need only consider this case since capacity can only be left unallocated in an optimal allocation pattern if $\sum_k \mu_k < M$, otherwise we can improve the value of the objective function by simply increasing the amount of allotted resources of one or more projects k for which $\hat{M}_k < \mu_k$.

When comparing two projects that both have non-decreasing ($\gamma \geq 1$) returns, the objective function is convex (see Eq. (1)). The maximum of a convex function defined on a closed convex set (such as the set of all solutions to our system of linear constraints) is achieved at an extreme point (Luenberger 2003). This implies that at least one of the projects will be appointed its boundary value, which one will depend on their respective total profits. A solution that allocates to more than one project with non-decreasing returns an intermediate amount of capacity is therefore dominated. □

Proof (Proposition 3): For the functions given previously (Proof Proposition 1), we derive the second-order partial derivative $\frac{\partial^2 E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]}{\partial \hat{M} \partial C} = \frac{\alpha\gamma}{2\beta(1-\gamma)} \left(\left(\frac{\hat{M}}{1+\beta}\right)^{\gamma-1} - \mu^{\gamma-1}\right)$ when $\gamma \neq 1$ and $\frac{\partial^2 E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]}{\partial \hat{M} \partial C} = \frac{\alpha}{2\beta} \ln\left(\frac{\mu(1+\beta}{\hat{M}}\right)$ when $\gamma = 1$. The sign of the second-order partial derivative $\frac{\partial^2 E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]}{\partial \hat{M} \partial C} \geq 0$. The analysis for α is analogous. □

Proof (Proposition 4): We can easily derive the second-order partial derivative from the first-order partial derivative given in Proof (Proposition 1) for $\gamma \neq 1$: $\frac{\partial^2 E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]}{\partial \hat{M} \partial \mu} = \frac{\alpha C \gamma}{2\beta} \mu^{\gamma-2} \geq 0$. When $\gamma = 1$, $\frac{\partial^2 E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]}{\partial \hat{M} \partial \mu} = \frac{\alpha C}{2\beta\mu} \geq 0$. □

Proof (Proposition 5): From the first-order partial derivative in Eq. (9), we obtain that

$$\begin{aligned}&\text{If } \gamma \neq 1 \Rightarrow \\ &\left\{ \frac{\partial^2 E[y(\hat{M})|\hat{M}\geq\mu(1-\beta)]}{\partial\hat{M}\partial\beta} \quad = \quad \frac{\alpha\gamma C}{2\beta^2(1-\gamma)}\Big(\mu^{\gamma-1} - \hat{M}^{\gamma-1}(1+\beta)^{-\gamma}(1+\beta\gamma)\Big),\right. \\ &\text{if } \gamma = 1 \Rightarrow \\ &\left\{ \frac{\partial^2 E[y(\hat{M})|\hat{M}\geq\mu(1-\beta)]}{\partial\hat{M}\partial\beta} \quad = \quad \frac{\alpha C}{2\beta}\Big(- 1/\beta \ln\Big(\frac{\mu(1+\beta)}{\hat{M}}\Big) + \frac{1}{1+\beta}\Big).\right.\end{aligned}$$

The sign of these second order derivatives is negative for all values below M^*, defined in Proposition 5, and positive for all values above this value. Based on these findings, one can easily distinguish the intervals in which $\frac{\partial^2 E[y(\hat{M}_1)]}{\partial\hat{M}\partial\beta}$ and $\frac{\partial^2 E[y(\hat{M}_2)]}{\partial\hat{M}\partial\beta}$ have the same sign. □

Proof (Proposition 6): If $\gamma \leq 1$, the project's profit is concavely increasing in $\hat{M}$. The optimal capacity can therefore be obtained from Eq. (7), with $\frac{\partial E[y(\hat{M})|\hat{M}=m_1]}{\partial\hat{M}} = \alpha C\gamma m_1^{\gamma-1}$ and $\lambda = \alpha_{ROI}$. We determine m_1 from $\alpha_{ROI} = \alpha C\gamma m_1^{\gamma-1}$. For $\gamma > 1$, we have shown in the proof of Proposition 2 that the maximum is achieved at an extreme point, and thus the project will be allocated a boundary amount of capacity. When the average profit per capacity unit of the project exceeds α_{ROI} this boundary amount is μ and otherwise we allocate $\underline{L} = 0$. □

Proof (Proposition 7): From Proposition 6 it follows that $\frac{\partial y(m_1)}{\partial\alpha_{ROI}} = \frac{1}{\gamma-1}\left(\frac{\alpha_{ROI}}{\gamma C\alpha}\right)^{1/(\gamma-1)}$ < 0; and $\frac{\partial^2 y(m_1)}{\partial^2\alpha_{ROI}} = \frac{1}{\gamma C\alpha(\gamma-1)^2}\left(\frac{\alpha_{ROI}}{\gamma C\alpha}\right)^{(2-\gamma)/(\gamma-1)} > 0$. □

Proof (Proposition 8): This proposition immediately follows from the results in Table 2.□

Proof (Proposition 9): We established in Proposition 1 that $E[y(\hat{M})|\hat{M} \geq \mu(1-\beta)]$ is concavely increasing in $\hat{M}$ for all values of γ. The optimal capacity can therefore be obtained from Eq. (7). The partial derivatives $\frac{\partial E[y(m_1)|m_1\geq\mu(1-\beta)]}{\partial\hat{M}}$ and $\frac{\partial E[y(m_2)|m_2\geq\mu(1-\beta)]}{\partial\hat{M}}$ are obtained from Eq. (9). We determine m_1 from $\alpha_{ROI} = \frac{\alpha C\gamma}{2\beta(1-\gamma)}\left(m_1{}^{(\gamma-1)}(1+\beta)^{(1-\gamma)} - \mu^{(\gamma-1)}\right)$ and m_2 from $\alpha_{ROI} = \frac{\alpha C}{2\beta}\ln\left(\frac{\mu(1+\beta)}{m_2}\right)$. □

Proof (Proposition 10): From Proposition 9, it follows that $\frac{\partial E[y(m_1)|m_1\geq\mu(1-\beta)]}{\partial\alpha_{ROI}} = -(1+\beta)\frac{\alpha_{ROI}2\beta}{C\alpha\gamma}\left(\frac{\alpha_{ROI}2\beta(1-\gamma)}{C\alpha\gamma} + \mu^{\gamma-1}\right)^{\frac{2-\gamma}{\gamma-1}}$. If $m_1 > 0$ the last factor of the partial derivative is also larger than zero because of their similar composition. It is clear that $\frac{\partial E[y(m_1)|m_1\geq\mu(1-\beta)]}{\partial\alpha_{ROI}} < 0$. The second derivative is $\frac{\partial^2 E[y(m_1)|m_1\geq\mu(1-\beta)]}{\partial^2\alpha_{ROI}} = (1+\beta)\frac{2\beta}{C\alpha\gamma}\left(\frac{\alpha_{ROI}2\beta(1-\gamma)}{C\alpha\gamma} + \mu^{\gamma-1}\right)^{\frac{3-2\gamma}{\gamma-1}}\left(\frac{\alpha_{ROI}2\beta}{C\alpha\gamma} - \mu^{\gamma-1}\right)$. When $m_1 \geq 0$, the sign of the second partial derivative is determined by the last factor. So that $\frac{\partial^2 E[y(m_1)|m_1\geq\mu(1-\beta)]}{\partial^2\alpha_{ROI}} < 0$ if $\mu^{\gamma-1} > \frac{\alpha_{ROI}2\beta}{C\alpha\gamma}$; this is always the case when $\gamma > 1$

(since $m_1 > 0$ and the similarities in composition). If $\gamma < 1$, the second partial derivative is smaller than zero as long as $\alpha_{ROI} < \frac{C\alpha\gamma}{2\beta}\mu^{\gamma-1}$.

From Proposition 9, it follows that $\frac{\partial E[y(m_2)|m_2 \geq \mu(1-\beta)]}{\partial \alpha_{ROI}} = -\mu(1+\beta)\exp\left(-\frac{\alpha_{ROI}2\beta}{\alpha C}\right)\left(\frac{\alpha_{ROI}2\beta}{\alpha C}\right) < 0$. The second derivative becomes $\frac{\partial^2 E[y(m_2)|m_2 \geq \mu(1-\beta)]}{\partial^2 \alpha_{ROI}} = \mu(1+\beta)\exp\left(\frac{\alpha_{ROI}2\beta}{\alpha C}\right)\left(\frac{2\beta}{C\alpha}\right)\left(\frac{\alpha_{ROI}2\beta}{\alpha C} - 1\right)$. From this we derive that for $\alpha_{ROI} < \frac{C\alpha}{2\beta}$, $\frac{\partial^2 E[y(m_2)|m_2 \geq \mu(1-\beta)]}{\partial^2 \alpha_{ROI}} < 0$ and for $\alpha_{ROI} > \frac{C\alpha}{\beta}$: $\frac{\partial^2 E[y(m_2)|m_2 \geq \mu(1-\beta)]}{\partial^2 \alpha_{ROI}} > 0$. □

Proof (Proposition 11): Replacing m_1 and m_2 with $\mu(1+\beta)$ in Table 3 delivers the minimal values for the return on investment, which is zero. □

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